Phenomenological lattice model of the high-temperature phase transition in quantum chromodynamics

Chu Xin Chen and Carleton DeTar
Department of Physics, University of Utah, Salt Lake City, Utah 84112
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We propose a simple phenomenological lattice model for the high-temperature phase transition in a quark-gluon plasma, in which the elementary dynamical variables are Wilson lines and SU(N) chiral spins. A mean-field analysis of the model shows a second-order chiral phase transition for $N=2$ and a first-order phase transition for $N=3$. We explore the phase diagrams obtained in the mean-field approximation by varying the several parameters of the model, including a mass term that breaks SU(3)×SU(3), leaving SU(2)×SU(2). Our study admits the possibility that the high-temperature phase transition in QCD is a chiral-symmetry-restoring phase transition and not a "deconfinement" phase transition.

I. INTRODUCTION

The behavior of quantum chromodynamics (QCD) at high temperatures (a few hundred MeV) has attracted much interest in recent years, because of its possible relevance to the evolution of the early Universe and because it may be possible to develop such temperatures in heavy-ion collisions.[1] Although at extremely high temperatures (several GeV), strongly interacting matter is well described over distances of the order of the inverse temperature as a quasifree plasma of quarks and gluons, easily treated in perturbation theory, the long-range structure at high temperature and the lower-temperature properties are not so simple. Indeed, it has been suggested that the plasma is, in fact, dynamically confined, and that its low-lying, long-range modes of excitation are color singlets at all temperatures.[2]

In the absence of experimental information, much of what we know about the structure of the QCD plasma comes from numerical simulations of lattice versions of QCD. From these simulations, we believe that there is a deconfinement phase transition in a gluon plasma that is second order for SU(2) color and first order for SU(3) color.[3] The Wilson line is the order parameter for that phase transition. When dynamical quarks are introduced, the Wilson line is no longer a rigorous order parameter and the deconfinement phase transition is weakened or obliterated. The smaller the bare quark mass, the more pronounced the weakening effect.[4]

If the quarks are massless and occur in an SU(N) flavor multiplet, QCD has an SU(N)×SU(N)×U(1)×U(1)$_A$ symmetry. The SU(N)×SU(N) chiral symmetry is spontaneously broken at low temperatures. On the other hand, the U(1)$_A$ (axial) symmetry is explicitly broken by the gauge anomaly.[5] From analytic studies of lattice QCD, it is expected that at very high temperatures, both chiral symmetries are restored.[6] Since the gauge anomaly persists to high temperatures, it is possible that the restoration of the SU(N)×SU(N) symmetry is associated with a genuine phase transition, whereas the restoration of U(1)$_A$ is asymptotic. However, it has also been suggested that two chiral phase transitions could occur.[7] Numerical studies currently in progress should help to provide partial answers to these questions.[8]

Thus, on the one hand, the deconfining phase transition is weakened as the quark mass is decreased, and on the other, the chiral phase transition (let us assume it is a single phase transition) is weakened as the quark mass is increased. The relationship between these two phase transitions and the nature of the phase transition cannot be deduced from symmetry considerations alone. We have three possibilities: (1) The two phase transitions become entangled with each other, so that only one phase transition occurs at any bare quark mass, (2) there are two phase transitions at some quark masses, or (3) they are disjoint, so that at some quark mass, no phase transition occurs. Numerical simulations disfavor the second possibility.[9–11] Between the first and the third, the numerical simulations have disagreed.[10,11] However, recent high-statistics numerical simulations using the staggered-fermion scheme suggests that there is a first-order chiral phase transition at very small bare quark masses, but that there is no phase transition at slightly larger masses, corresponding to the third choice.[11] The simple model that we propose may help to clarify the relationship between the two phase transitions.

Although numerical simulations are helpful, they have so far been severly limited to the study of static properties of the plasma: namely, the measurement of screening lengths and bulk thermodynamic quantities. Current numerical techniques do not permit the study of real-time finite-temperature fluctuations and quasiequilibrium processes. An understanding of those processes is crucial to a theoretical analysis of heavy-ion collisions and cosmology. Moreover, the numerical simulations require expensive computation and produce results that can be explained only in numerical terms. Thus it is desirable to formulate simple phenomenological models of the plasma and of the phase transition with two goals in mind: first, to attempt to isolate the essential qualitative features of the phase transition, so that it can be understood in simpler analytical terms; second, to point the way to a phenomenological model that can be extended to a study of real-time—finite-temperature and quasiequilibrium processes. To this end we propose a simple phenomenological lattice model intended to characterize the important long-range nonhydrodynamic modes of excitation.

Our model is similar in spirit to the Landau-Ginzburg model of superconductivity; it introduces a minimal set of fields characterizing the important long-range modes, including the order parameters of the phase transition, and a minimal number of interaction terms sufficient to produce the phase transition. In its present version the model is formulated on a three-dimensional lattice, which can be considered a high-temperature approximation to a four-dimensional theory (although we do not derive it as such). The elementary fields are the Wilson line and an SU(N) chiral field. A minimal finite-temperature effective action is constructed that preserves the necessary symmetries and incorporates some understanding of lattice gauge theory, namely,

$$S_{\rm eff}=\frac{1}{2}\left\{\beta_w \sum_{(x,y)} [\mathrm{tr}W(x)][\mathrm{tr}W^\dagger(y)]+\beta_h \sum_x \mathrm{Re\,tr}W(x) -\beta_i \sum_{(x,y)} [\mathrm{Re\,tr}W(x)+\mathrm{Re\,tr}W(y)]\mathrm{tr}[U(x)U^\dagger(y)]+\beta_c \sum_{(x,y)} \mathrm{tr}[U(x)U^\dagger(y)]+\sum_x \mathrm{tr}[MU^\dagger(x)]\right\}+\text{c.c.}\,, \tag{1.1}$$

where the sum x is over a three-dimensional cubic lattice, the sum (x,y) is over unique nearest-neighbor pairs, $W(x)$ is an SU(3) color matrix for the Wilson line, U is an SU(N) flavor matrix for the chiral field, M is a constant mass matrix, β_w, β_h, β_i, and β_c are parameters determining the Wilson line self-coupling, the intrinsic "magnetic field," the Wilson-line—chiral-field interaction, and the chiral-field self-coupling, respectively, and the constant matrix M determines the "quark mass."

The Wilson-line part of the effective action follows the suggestion of Svetitsky and Yaffe,[12] and the chiral part of the effective action has been well studied.[13] The interaction term is new. With such a model it is possible to explore the interplay between the deconfining phase transition of the pure gluon plasma and the chiral phase transition of the chiral model. Our justification for this model is given in Sec. II. Some key features and limitations should be emphasized.

(i) The elementary fields are color singlets both below and above the phase transition. There are no quasiquark or quasigluon modes.

(ii) With the mass term present the model makes it possible to study the breakdown of an SU(3)×SU(3) chiral symmetry to SU(2)×SU(2) as the strange quark mass is turned on, and a complete breakdown of the flavor symmetry.

(iii) In this form the model describes the restoration of an SU(N)×SU(N) chiral symmetry, but not of a U(1) axial symmetry. However, by doubling the meson species, the model is easily extended to that case.

(iv) The model is intended to be useful only in the vicinity of the phase transition, and not at very low or very high temperatures.

There have been several other efforts to formulate an effective action for the QCD phase transition. Gocksch and Ogilvie,[14] following the techniques of the strong-coupling approximation of Kawamoto and Smit[15] and Kluberg-Stern, Morel, and Petersson[16] started with the Wilson formulation of lattice QCD and derived a high-temperature effective action quite similar to ours in spirit, formulated in terms of the Wilson line and an external meson source. They found two phase transitions: one deconfining and one chiral. Their model is rather cumbersome and, because it relies on strong-coupling approximations, depends sensitively on the lattice scheme (in their case the Wilson model) from which they began. On the other hand, all parameters of their model are determined, in principle, from the zero-temperature hadron spectrum. Our model seeks a simpler, more flexible, and, we hope, more useful formulation, although at the expense of introducing a set of parameters, whose quantitative variation with temperature is unknown.

In this work we describe an extensive mean-field calculation exploring the phase structure of the ensemble characterized by our effective action. The results are presented in Sec. III. Finally, we discuss possible extensions and applications of our model in the concluding section.

II. THE HIGH-TEMPERATURE EFFECTIVE ACTION

In this section we offer motivation for our effective action (1.1).

We restrict our attention to fields that are likely to excite the longest-range modes of the plasma in the vicinity of the phase transition. From studies of static screening lengths in the pure gluon plasma in SU(3) color,[17] it is known that the Wilson line couples strongly to the color-singlet plasmon mode, the longest-range screening mode in the pure gluon plasma. The effective action of Svetitsky and Yaffe[12] characterizes the phase transition of the pure gluon theory as a ferromagnetic phase transition that breaks an underlying $Z(3)$ symmetry. Their action is

$$S_w=\tfrac{1}{2}\beta_w \sum_{(x,y)} [\mathrm{tr}W(x)][\mathrm{tr}W^\dagger(y)]+\text{c.c.}\,, \tag{2.1}$$

where $W(x)$ is an SU(3) color matrix. The model has a $Z(3)$ symmetry under the transformation

$$W(x) \rightarrow W(x)e^{2\pi i n/3} \tag{2.2}$$

for $n=0,1,2$. When β_w is large, the $Z(3)$ symmetry is spontaneously broken and the Wilson line has a nonzero expectation value:

$$\langle W(x) \rangle = \overline{W} = wI , \tag{2.3}$$

where I is the unit 3×3 matrix. For small β_w the symmetry is restored and $w=0$. When quarks are introduced, the $Z(3)$ symmetry is broken in the four-dimensional functional integral by the fermion determinant.[12] In the effective action this phenomenon is modeled by introducing a term analogous to the coupling of an external "magnetic field" to the Wilson line "spin" variable, thus

$$S_w' = S_w + \beta_h \sum_x \mathrm{Re}\,\mathrm{tr} W(x) . \tag{2.4}$$

If β_h is large enough, the first-order phase transition is obliterated.

Of course, when quarks are introduced, it is also necessary to introduce dynamical modes formed from color-singlet combinations of the quarks. If the quarks are massless, QCD is chirally symmetric under $\mathrm{SU}(N) \times \mathrm{SU}(N) \times \mathrm{U}(1)_A$. Since the axial symmetry $\mathrm{U}(1)_A$ is explicitly broken by the gauge anomaly, we shall ignore that symmetry for the moment. We restrict our attention to the Goldstone modes and modes associated with them by symmetry. Since the model for the deconfinement phase transition (2.1) has been formulated on a lattice, we turn to a convenient high-temperature lattice formulation of chiral symmetry breaking, namely, the $\mathrm{SU}(N)$ chiral model:[13]

$$S_c = \tfrac{1}{2}\beta_c \sum_{(x,y)} \mathrm{tr}[U(x)U^\dagger(y)] + \mathrm{c.c.} , \tag{2.5}$$

where the sum (x,y), is over nearest-neighbor sites on a three-dimensional lattice and $U(x)$ is an $\mathrm{SU}(N)$ flavor matrix. The model has an $\mathrm{SU}(N)\times\mathrm{SU}(N)$ chiral symmetry under the transformation

$$U(x) \rightarrow AU(x)B^\dagger , \tag{2.6}$$

where A and B are $\mathrm{SU}(N)$ matrices. For small β_c, $\overline{U}=0$. For large β_c the $\mathrm{SU}(N)\times\mathrm{SU}(N)$ chiral symmetry is broken and the chiral order parameter $\overline{U}$ defined by

$$\langle U(x) \rangle = \overline{U} \tag{2.7}$$

has a nonzero value. Through a chiral rotation, it can always be arranged so that the mean value of U is diagonal. Because the action is Hermitian, it must be real. If one of the N quarks acquires a mass, the $\mathrm{SU}(N)\times\mathrm{SU}(N)$ symmetry is explicitly broken to $\mathrm{SU}(N-1)\times\mathrm{SU}(N-1)$. Including a mass term gives the model defined by

$$S_c' = S_c + \tfrac{1}{2} \sum_x \mathrm{tr}[MU^\dagger(x)] + \mathrm{c.c.} , \tag{2.8}$$

where M is a constant mass matrix that we choose to be diagonal.

We now seek to combine the Wilson line effective action (2.1) with the chiral action (2.8), including an interaction term between the chiral fields and the Wilson line. A minimal possibility for the coupling between the fields that preserves the $\mathrm{SU}(N)\times\mathrm{SU}(N)$ symmetry but not the $Z(3)$ symmetry involves a three-point coupling

$$S_{\mathrm{eff}} = S_w' + S_c' - \tfrac{1}{2}\beta_i \sum_{(x,y)} [\mathrm{Re}\,\mathrm{tr} W(x) + \mathrm{Re}\,\mathrm{tr} W(y)] \times \mathrm{tr}[U(x)U^\dagger(y)] - \mathrm{c.c.} \tag{2.9}$$

We have also considered higher-order couplings, but choose to limit our attention to this simplest possibility. Putting all of the terms together gives the effective action

$$S_{\mathrm{eff}} = \frac{1}{2}\left[\beta_w \sum_{(x,y)} [\mathrm{tr} W(x)][\mathrm{tr} W^\dagger(y)] + \beta_h \sum_x \mathrm{Re}\,\mathrm{tr} W(x) - \beta_i \sum_{(x,y)} [\mathrm{Re}\,\mathrm{tr} W(x) + \mathrm{Re}\,\mathrm{tr} W(y)] \mathrm{tr}[U(x)U^\dagger(y)] + \beta_c \sum_{(x,y)} \mathrm{tr}[U(x)U^\dagger(y)] + \sum_x \mathrm{tr}[MU^\dagger(x)] \right] + \mathrm{c.c.} \tag{1.1}$$

We offer the following arguments justifying our choice of the interaction term (2.9).

(i) The Wilson line can be expanded in terms of the color scalar potential ϕ_μ^a as follows:

$$\mathrm{tr} W(x) = \mathrm{tr}\exp(ig\phi_0^a\lambda^a/T) \approx 3 - \tfrac{3}{2}g^2\phi_0{}^2/T^2 , \tag{2.10}$$

where T is the temperature, g the QCD coupling, and λ^a the eight generators of SU(3). The Wilson line portion of the action can be expressed in terms of color-singlet polynomials in the scalar potential. These polynomials are composite field operators for various "glueball" modes. Thus the three-point coupling includes a coupling between a composite "glueball" source field

$$G = (\phi_0^a)^2 \tag{2.11}$$

and the Goldstone boson. In other words, the expected decay of the scalar glueball into a pair of pions is incorporated into the model by this interaction.

(ii) Consider a four-dimensional lattice gauge theory with quarks and gluons. The meson field interaction in (2.5) corresponds to a quark-antiquark pair hopping from one site to the nearest-neighbor site in the lattice, as shown in Fig. 1(a). At finite temperature, this hopping term is modified by paths that allow either the quark or the antiquark to loop around in the imaginary-time direction once, as shown in Fig. 1(b). The antiperiodic boundary condition for the fermion gives a negative sign for

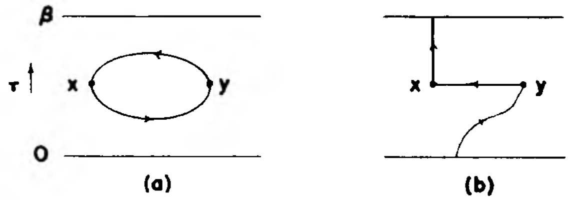

FIG. 1. Euclidean-space quark line diagrams for the meson (chiral field) propagator. The imaginary-time variable τ runs from 0 to β, the inverse temperature. (a) Direct contribution. (b) A thermal contribution giving rise to a chiral-field—Wilson-line coupling.

this correction to the meson hopping term: hence the sign in (2.9). This argument is only heuristic, since the hopping-parameter expansion is certainly no good for zero-mass fermions.

(iii) It has been argued by some that the "constituent-quark mass" is large when chiral symmetry is spontaneously broken and small when it is restored. Now the "magnetic field" term in (2.4) should be reduced when the constituent-quark mass increases. Viewing the interaction term (2.9) as a correction to the magnetic field term (2.4), we see that the correction is large when chiral symmetry is spontaneously broken and U has a large expectation value, and it is small when chiral symmetry is restored and U has a small expectation value. With the minus sign in (2.9) the correction has the desired effect.

We conclude with a few remarks about our effective action.

(i) It is important that we keep both the interaction term and the "magnetic field" term (1.1) in our effective action. The interaction term gives a correction to the magnetic field strength that depends on the chiral order parameter:

$$\beta_{h,\mathrm{eff}}=\beta_h-\beta_i z\,\mathrm{tr}(\overline{U}^{\,2})\ , \tag{2.12}$$

where $z=6$ is the number of nearest neighbors. If the term β_h were not present, the interaction term would develop an unphysical negative effective magnetic field.

(ii) With a negative interaction term, the deconfinement phase transition can trigger chiral-symmetry restoration. This can be understood as follows: from the point of view of the chiral phase transition, decreasing the nearest-neighbor interaction β_c promotes restoration of the chiral symmetry. The effective nearest-neighbor interaction is given by

$$\beta_{c,\mathrm{eff}}=\beta_c-6\beta_i w \tag{2.13}$$

for three colors. If the Wilson-line expectation value suddenly increases, the interaction term causes the effective nearest-neighbor chiral coupling to decrease suddenly, thereby promoting a restoration of the chiral symmetry.

(iii) Because we are unable to derive this effective action rigorously from a lattice theory of QCD, we cannot set values for the parameters or for their variation with temperature or bare quark mass. Nevertheless, we require that as temperature is increased, β_w increases, and β_c decreases in accordance with what we expect from the uncoupled theories (2.1) and (2.5).

III. MEAN-FIELD CALCULATION

Mean-field theory provides a quick glance at the qualitative features of the lattice model (1.1). Monte Carlo simulations could also be readily carried out for such an action in analogy with the calculations that have already been done for the uncoupled theories.[18,19] In both cases the Monte Carlo results agree well with the mean-field calculation, although there are always some lingering questions about the validity of the mean-field approach.[7] Given the large dimension of the parameter space, it is appropriate to start with the simplest method to explore the important features of the model.

In terms of the mean value of the Wilson line $\overline{W}$, and the chiral spin $\overline{U}$, the single site action is given by

$$\begin{aligned}S_1(W,U,\overline{W},\overline{U})=\frac{1}{2}[&(3\beta_w wz+\beta_h)\mathrm{tr}W\\&-\beta_i z(\mathrm{Re\,tr}W+3w)\mathrm{tr}(\overline{U}^{\,\dagger}U)\\&+\beta_c z\,\mathrm{tr}(\overline{U}^{\,\dagger}U)+\mathrm{tr}(MU)]+\mathrm{c.c.}\ ,\end{aligned} \tag{3.1}$$

where $z=6$ is the number of nearest neighbors and the mean values satisfy the self-consistency relations:

$$\overline{W}=\langle W\rangle=wI,\quad \overline{U}=\langle U\rangle\ . \tag{3.2}$$

The expectation values are defined by

$$\begin{aligned}\langle O\rangle&=\int DW\,DU\,O\exp[S_1(W,U,\overline{W},\overline{U})]/Z(\overline{W},\overline{U})\ ,\\ Z(\overline{W},\overline{U})&=\int DW\,DU\exp[S_1(W,U,\overline{W},\overline{U})]\ .\end{aligned} \tag{3.3}$$

The integration over the group manifold can be simplified using the general result[20]

$$\begin{aligned}Z(P)&=\int dU\exp[\mathrm{tr}(PU^\dagger)+\mathrm{tr}(P^\dagger U)]\\&=2\sum_{jkln=0}^{\infty}\frac{1}{(j+2k+3l+n+2)!(k+2l+n+1)!}\\&\qquad\times\frac{x^j}{j!}\frac{y^k}{k!}\frac{z^l}{l!}\frac{\Delta^n}{n!}\ ,\end{aligned} \tag{3.4}$$

where

$$x=\mathrm{tr}(PP^\dagger),\quad y=\tfrac{1}{2}\{[\mathrm{tr}(PP^\dagger)]^2-\mathrm{Tr}[(PP^\dagger)^2]\}\ ,$$

$$z=\det(PP^\dagger),\quad \Delta=\det P+\det P^\dagger\ .$$

If $P=pI$ is a multiple of the identity, then we have[19]

$$Z(P)=\sum_{m=-\infty}^{\infty}\det I_{m+j-i}(2p)\ , \tag{3.5}$$

where I_n is the modified Bessel function and the determinant is taken of the matrix formed by varying the row and column indices i and j over $[1,N]$. If the integration over the chiral spin is carried out first, the calculation of the partition function and expectation values is reduced to an integration over the Wilson line trace, i.e., an integration over the classes of SU(3), which we do using the Weyl parametrization.

When there is no explicit chiral-symmetry-breaking mass term, then it is always possible to arrange so that $\overline{U}$ is a multiple of the identity:

$$\overline{U}=uI \ . \tag{3.6}$$

The order parameter is u. The self-consistency conditions (3.2) become

$$3w=\langle \mathrm{tr} W\rangle, \quad Nu=\langle \mathrm{tr} U\rangle \ . \tag{3.7}$$

For three flavors, we consider breaking the $SU(3)\times SU(3)$ symmetry to $SU(2)\times SU(2)$ by introducing a mass term (2.8) with

$$M=\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_s \end{vmatrix} \ . \tag{3.8}$$

In that case the mean value of the chiral spin can be arranged to have the form

$$\overline{U}=\begin{vmatrix} u_1 & 0 & 0 \\ 0 & u_1 & 0 \\ 0 & 0 & u_2 \end{vmatrix} \tag{3.9}$$

and u_1 is the order parameter for the residual $SU(2)\times SU(2)$ chiral symmetry. The self-consistency condition then becomes

$$\langle U_{11}+U_{22}\rangle=2u_1 \ . \tag{3.10}$$

The remaining integrations were carried out numerically on a VAX 11/785 computer and the self-consistency conditions were imposed in an iterative fashion for a range of choices of the coupling constants.

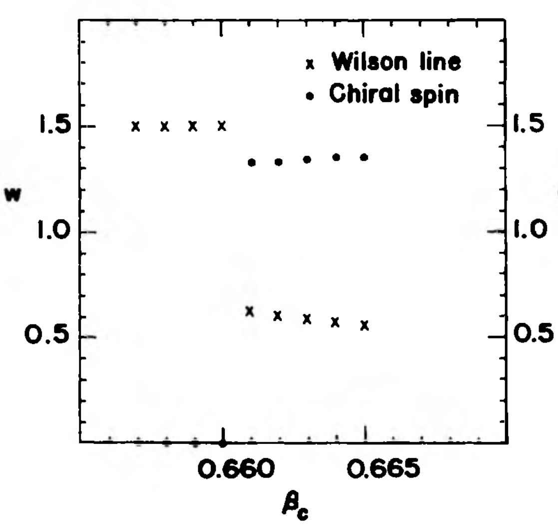

FIG. 2. A first-order phase transition revealed in the mean-field behavior of the Wilson-line and chiral-spin expectation values as a function of the chiral coupling β_c at fixed values $\beta_w=0.16$, $\beta_h=1.0$, $\beta_i=0.11$.

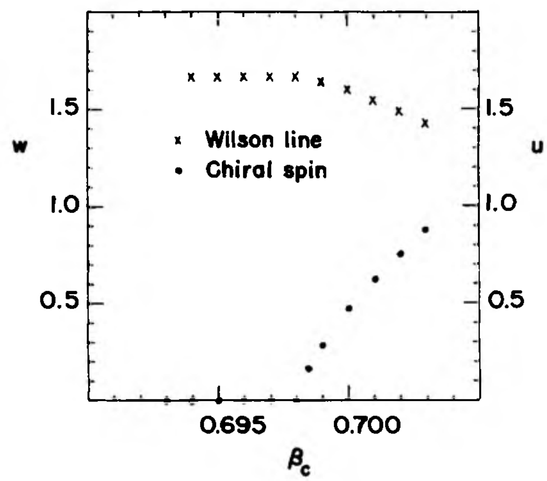

FIG. 3. A second-order phase transition. The parameters $\beta_w=0.16$, $\beta_h=1.0$, $\beta_i=0.11$ are fixed.

IV. RESULTS

A. Determining the order of the phase transition

We present results for the phase structure of our effective action in mean-field theory for SU(2) and SU(3) flavor with and without various quark mass terms. The phase structure is determined by observing the variation of the Wilson line w and chiral expectation value u as a function of the various parameters in the action. We attempt to distinguish a first-order from a second-order phase transition by looking for a discontinuity in the expectation values. In Figs. 2 and 3 we illustrate behavior that we classify as indicating first-order and second-order phase transitions and in Fig. 4 as no phase transition. Some small comptuational uncertainties arise in determining precisely where a first-order phase boundary gives way

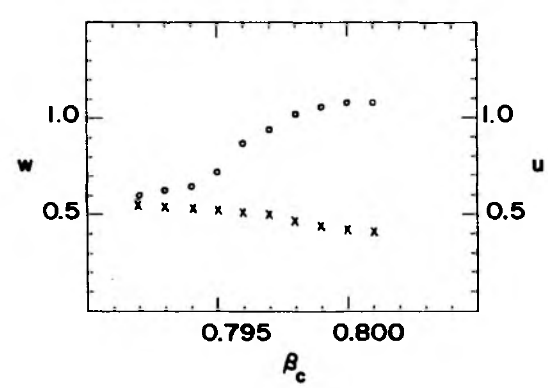

FIG. 4. No phase transition at $\beta_w=0.1$, $\beta_h=1.0$, $\beta_i=0.1$, $m=0.17$.

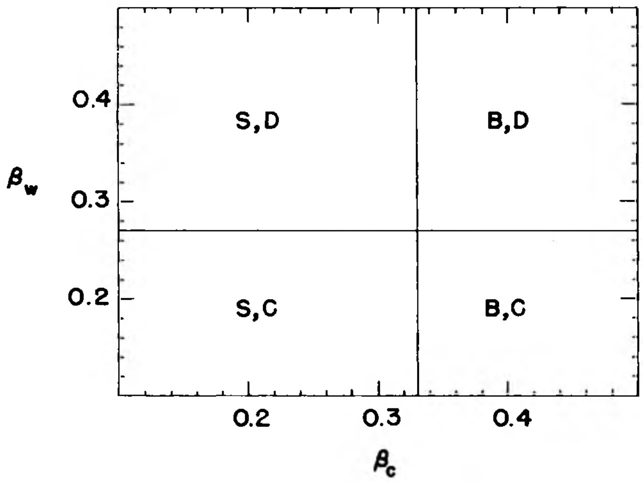

FIG. 5. The SU(2) β_w-β_c phase diagram with *no* interaction between the chiral and Wilson-line fields. The fixed parameters are $\beta_i=0$, $\beta_h=0$, and $M=0$. The lines denotes the phase boundaries. The phase attributes are labeled as follows: S, with manifest chiral symmetry; B, with spontaneously broken symmetry; C, confined; and D, deconfined.

to a second-order phase boundary, since at a junction between the two types of behavior, the discontinuity in the expectation values goes to zero.

B. Chiral SU(2): Noninteracting theory

We turn now to our results for chiral SU(2). We examine the phase diagram in the β_w-β_c plane, keeping β_h and β_i fixed. As a preliminary check of the computation, we turned off the interaction term by setting $\beta_i=0$. In this case the effective action becomes a sum of the Wilson-line action (2.4) and the chiral action (2.8), and we expect that the phase boundaries in the β_w-β_c plane are parallel to the β_w and β_c axes. We verified that the chiral phase transition occurred at the correct value of β_c and the Wilson-line phase transition occurred at the correct value of β_w with $\beta_h=0$, as shown in Fig. 5. Notice that with β_h sufficiently large, there is no deconfinement phase transition, and we obtain the phase diagram of Fig. 6.

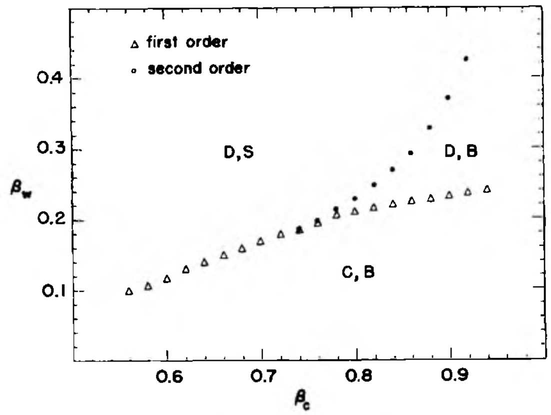

FIG. 7. The SU(2) β_w-β_c phase diagram *with* interaction between the chiral and Wilson-line fields. The fixed parameters are $\beta_i=0.12$ and $\beta_h=1$. The dots indicate a second-order phase transition. The triangles, first order. The phase attributes are labeled as in Fig. 5.

C. Chiral SU(2): interacting theory

As β_i is turned on, the Wilson line and chiral actions are coupled. The effective magnetic field term for the Wilson line varies, according to Eq. (2.12). In principle, it is possible to obtain an unphysically negative effective magnetic field, whereupon the Wilson-line expectation value becomes unphysically negative (i.e., yielding a complex free energy for a test quark). We avoided these unphysical regions of the parameter space. To do so required introducing a non-negligible value of β_h in the chirally broken phase. Now consider holding this value and that of β_i fixed while varying β_w and β_c. We get the phase diagrams shown in Figs. 7 and 8. Figure 7 corre-

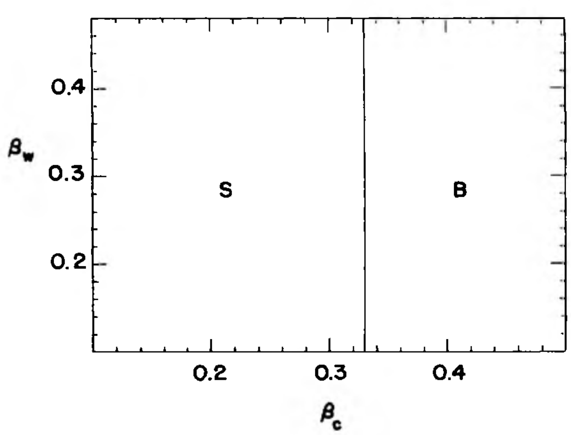

FIG. 6. The same as in Fig. 5, but with $\beta_h=1$.

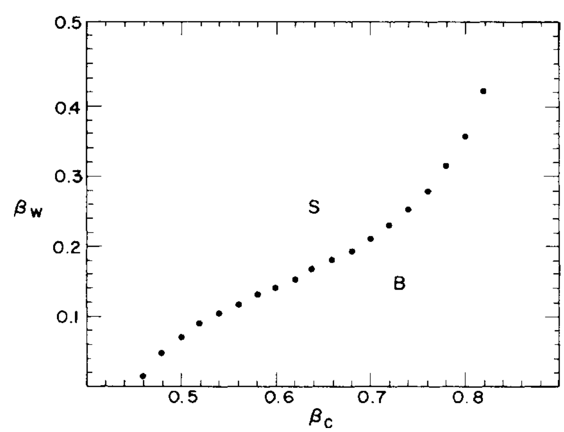

FIG. 8. The same as in Fig. 7, but with $\beta_i=0.10$, showing the obliteration of the deconfinement phase transition.

sponds to a stronger coupling between the Wilson line and chiral fields, and Fig. 8 corresponds to a weaker coupling. Notice that for the larger value of β_i the phase boundaries form a Y shape with first- or second-order phase transitions occurring on the branches. From the behavior of the Wilson-line and chiral field expectation values, we interpret the branch extending to the right as a vestigial first-order deconfining transition, the branch extending upwards as a vestigial second-order chiral transition, and the branch extending down to the left a combined chiral, deconfining phase transition (cf. Figs. 5 and 6). For the smaller value of β_i there is no remnant of the deconfining phase transition. The manner in which the deconfining phase transition disappears is interesting. We find that as β_i is decreased, the branch representing the deconfining phase transition shrinks in length until there is just a small region along the chiral phase boundary where a first-order phase transition occurs (not shown). An explanation of the disappearance of the deconfining phase transition is offered below.

We have shown that one effect of the interaction is to restrict the deconfinement phase transition to a limited range of parameter values. We explain this result as follows: As can be seen from (2.12), in regions of the parameter space with a large chiral expectation value, the effective magnetic field is small, whereas in regions with a small chiral expectation value, the effective magnetic field is large. Since a large effective magnetic field destroys the confinement/deconfinement phase transition, whereas a small value permits it to occur, we expect that the confinement/deconfinement phase transition is more likely to occur, if at all, when chiral symmetry is spontaneously broken, namely, at larger values of β_c. Moreover, when β_h is sufficiently large and β_i is sufficiently small, the deconfinement phase transition is not found at all.

D. SU(3) chiral model

A similar analysis for the SU(3) chiral model yields the phase diagrams shown in Figs. 9 and 10. Here the chiral phase transition is first order, the vestigial deconfinement phase transition again occurs only in the chirally broken phase, and the deconfining phase transition disappears for small β_i.

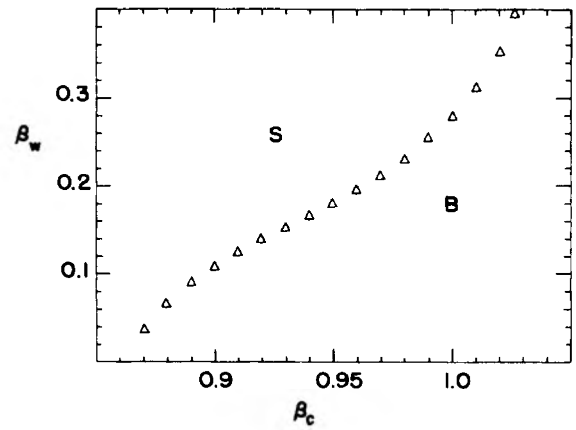

FIG. 10. The same as in Fig. 9, but with $\beta_i=0.05$, showing the obliteration of the deconfinement phase transition.

E. Introducing a strange quark mass

Introducing the mass term (2.8) with M as in (3.8) breaks the SU(3)$\times$SU(3) chiral symmetry explicitly to SU(2)$\times$SU(2). Since the chiral phase transition is first order in SU(3)$\times$SU(3) and second order in SU(2)$\times$SU(2), we expect that for a sufficiently large strange-quark mass the first-order phase transition gives way to a second-order phase transition. Indeed, this is the case, as illustrated in Figs. 11 and 12. In Fig. 11 we plot the phase diagram in β_c-m_s space, holding β_w, β_h, and β_i fixed. The effect of the mass term on the β_w-β_c phase structure is shown in Fig. 12. The changeover to a second-order

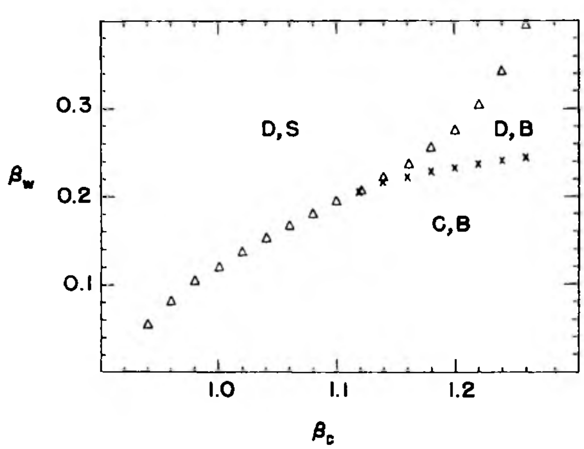

FIG. 9. The SU(3) β_w-β_c phase diagram with fixed parameters, $\beta_i=0.1$, $\beta_h=1$, and $M=0$. The triangles and crosses denote first-order phase boundaries.

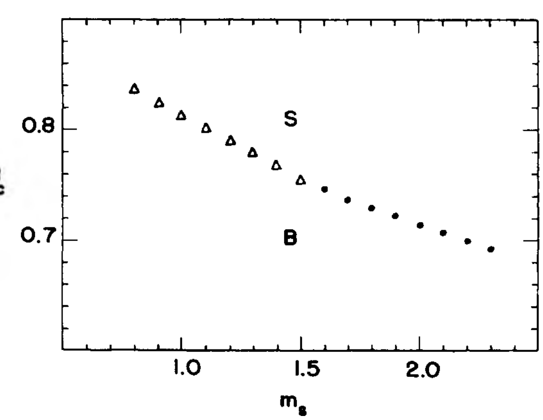

FIG. 11. Changeover from a first-order to a second-order chiral phase transition as a function of a strange-quark mass m_s: Phase diagram in β_c and m_s, with fixed parameters $\beta_w=0.1$, $\beta_h=1.0$, $\beta_i=0.1$. The triangles denote a first-order phase boundary and the dots second order.

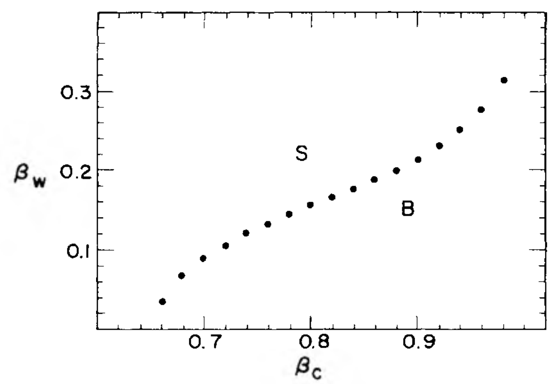

FIG. 12. Effect of a strange-quark mass on the β_w-β_c phase diagram with fixed parameters, $\beta_l = 0.1$, $\beta_h = 1.0$, $m_s = 2.0$.

phase transition occurs at a substantial value of β_m, suggesting that the first-order phase transition is quite stable against such perturbations. It is interesting to compare the phase diagrams in Figs. 9 and 12. The only parameter change in going from Fig. 9 to Fig. 12 is the strange-quark mass. We see that all other things being equal, introducing a substantial strange-quark mass converts the chiral phase transition from first order to second order and removes the deconfinement phase transition.

F. Introducing an SU(3)-symmetric quark mass

Introducing the mass term (2.8) with

$$M = mI \tag{4.1}$$

breaks the SU(3) chiral symmetry completely. At sufficiently large values of m we expect the first-order phase transition to disappear altogether. Our results are shown in Fig. 13. The expected result is found, but notice that

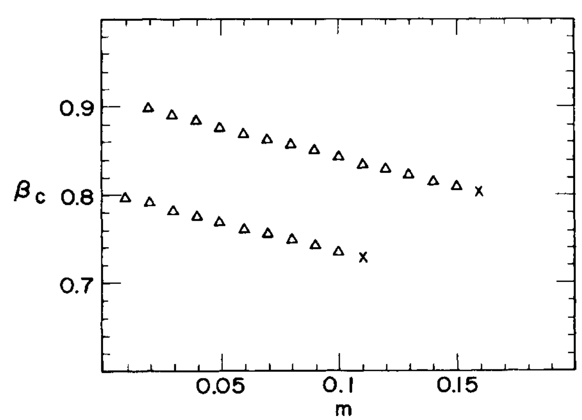

FIG. 13. Obliteration of the chiral phase transition as a function of an SU(3)-symmetric mass m. Two values of β_l are represented: the lower boundary corresponds to $\beta_l = 0.0$ and the upper to $\beta_l = 0.1$. The other fixed parameter values are $\beta_w = 0.1$ and $\beta_h = 1.0$.

the phase transition is obliterated for $m > 0.15$, a value considerably smaller than what was required for the strange-quark mass to convert the phase transition from first to second order, suggesting that the first-order phase transition is considerably less stable against such perturbations. In this respect, our results are in qualitative agreement with recent numerical simulations.[11]

V. SUMMARY AND DISCUSSION

We have propose a simple phenomenological effective lattice action for the high-temperature QCD phase transition, incorporating the Wilson-line and chiral fields. The model makes it possible to study the interaction of the two phase transitions in a larger parameter space than would be accessible in ordinary QCD. We have constructed the key portions of the phase diagrams in the multidimensional parameter space.

What have we learned about the nature of the phase transition? The physical parameters of QCD are the temperature and the bare quark masses. Our parameter space is much larger. As we have stressed above, even if our model is accepted as a valid representation of QCD, we are not able to fix our parameter values, nor the variation of our parameters with temperature and mass, except in broad qualitative terms. There are some important qualitative consequences of our analysis. We find that there are at least three phases that are distinguishable for some range of parameters: namely, a confined, chirally broken phase, a deconfined, chirally broken phase, and a deconfined, chirally restored phase. However, there is a path through our parameter space that connects all of these phases without crossing a phase boundary. This fact suggests that the high-temperature and low-temperature phases of QCD have some common qualitative features, e.g., the existence of color-singlet modes of excitation.[2]

As for the possible scenarios for the phase transition listed in the Introduction, all three are represented here: (1) a single combined phase transition occurs in Fig. 9 along the lower left branch, (2) two phase transitions occur (always with $T_{\text{deconf}} < T_{\text{chiral}}$), if an increase in temperature causes the parameter values to follow a path crossing the rightward branch and then the upward branch of Fig. 9, and (3) an exclusively chiral phase transition occurs in Fig. 10. We can learn more if we now turn to the numerical simulations. The results of Ref. 11 argue for the third possibility and Fig. 10, according to the following sequence of events as the SU(3)-symmetric quark mass is increased: As we have seen, a relatively small value of this mass destroys the chiral phase transition. An increase in quark mass results in a decrease in the effective magnetic field β_h. However, the small increase in quark mass required to obliterate the chiral phase transition is not expected to produce a dramatic decrease in the magnetic field. Therefore, it is easily possible in our model that over an intermediate range of quark masses, no phase transition occurs. As the quark mass is increased significantly, the effective magnetic field drops, and the deconfinement phase transition appears.

According to our findings, the following circumstances favor a purely chiral phase transition in QCD at zero quark mass: a weak coupling β_l, a relatively large effective magnetic field β_h, associated with a small quark mass, and a relatively large strange-quark mass.

Note added. While this report of our calculations was in preparation, we received a report by Akio Hosoya[21] in which a quite similar model is discussed. The models differ in that Hosoya introduces a four-point coupling between the Wilson-line and chiral fields, whereas we introduce a three-point coupling. Hosoya studies the minima of his effective potential, whereas we undertake a lowest-order mean-field analysis.

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